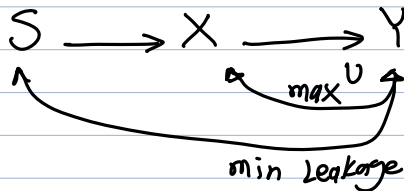


Recap



$P_{Y|X}$

$$I(S; Y) = \sum_s \sum_y P(s, y) \log \frac{P(s, y)}{P(s)P(y)}$$

$$\frac{P(s, y)}{P(s)P(y)}$$

$$\frac{P(y|s)}{P(y)} = \frac{P(s|y)}{P(s)}$$

$$\max_{s, y} \left| \log \frac{P(s|y)}{P(s)} \right| \leq \epsilon$$

LIP

$$\max \log \frac{P(s|y)}{P(s)} \leq \epsilon_0$$

$$\min \log \frac{P(s|y)}{P(s)} \geq -\epsilon_1$$

$\epsilon_0, \epsilon_1 > 0$

DP

$x \in X^n$
 $\equiv \equiv \equiv$
 # indiv. element

$x' \in X^n$
 $\equiv \equiv \equiv$

$x \sim x'$
 diff in one element

$$M: X^n \rightarrow Y$$

$$M(x)$$

$$M(x')$$



$$Pr(M(x) = y) \stackrel{\epsilon}{\approx} Pr(M(x') = y)$$

ϵ -DP

$$\left| \log \frac{Pr(M(x) = y)}{Pr(M(x') = y)} \right| \leq \epsilon$$

$$\left| \log \frac{Pr(y|x)}{Pr(y|x')} \right| \leq \epsilon$$

$\forall x, x': x \sim x'$

(ϵ, δ)

$\forall E \subseteq Y$

$\forall x, x' \in X^n$

$$Pr(M(x) \in E) \leq e^\epsilon Pr(M(x') \in E) + \delta$$

$\delta = \cdot$

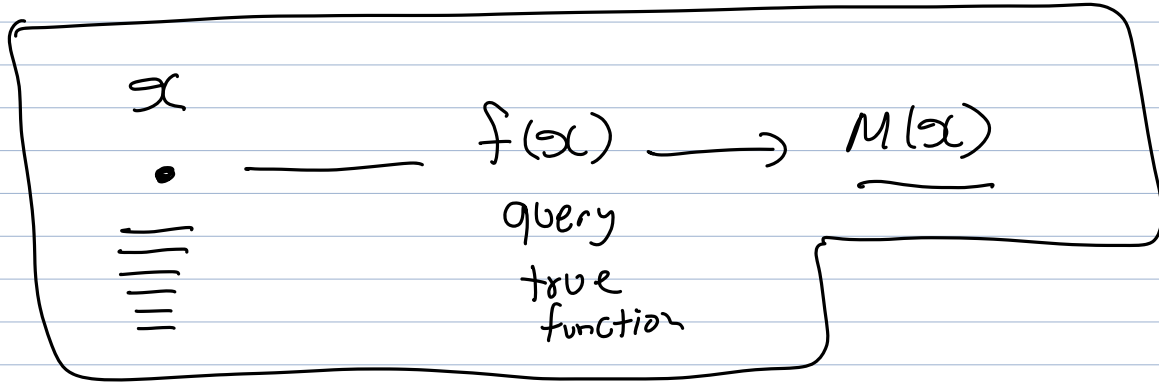
n

$\delta \ll \frac{1}{n}$
 $\delta = 10^{-11}$

ϵ -local-DP

$$\left| \log \frac{Pr(Y|\alpha)}{Pr(Y|\alpha')} \right| \leq \epsilon \quad \forall y, \alpha, \alpha'$$

Stronger DP



$\alpha: 111$ $\text{maj}(111) = 1$
 $\alpha': 122$ $\text{maj}(122) = 2$

$\left\{ \begin{array}{l} Pr(M(111) = 1) \\ Pr(M(111) = 2) \end{array} \right.$

$\alpha \in \{0,1\}^n$ $f(\alpha) = \sum \alpha_i$ sum function
 α'

$M(\alpha) = \sum \alpha_i + N$
 $N \sim \log\left(\frac{\Delta f}{\epsilon}\right)$
 ϵ -DP $\Delta f = 1$

$S \rightarrow X = f(S) \rightarrow Y$
 $S = (000)$ $X = \sum s_i$

$S' = (110)$

$$\left| \log \frac{P(y|s)}{P(y|s')} \right| \leq \epsilon \quad \forall y \quad \forall s, s': s \sim s'$$

$$X \rightarrow f(x) \rightarrow M(x)$$

$P_{s,x}$

$$\frac{P(y|s)}{P(y|s')} = \frac{P(s|y)P(y)}{P(s)} \cdot \frac{P(s')}{P(s'|y)P(y)}$$

$\uparrow \leq e^{\epsilon_U}$ $\uparrow \leq e^{\epsilon_L}$

$$e^{-\epsilon} \leq \frac{P(y|s)}{P(y|s')} \leq e^{\epsilon} \Rightarrow e^{-\epsilon} \leq \frac{P(s|y)}{P(s'|y)} \leq e^{\epsilon} \frac{P(s)}{P(s')} \quad \forall s, s'$$

Relations :

$$\left. \begin{array}{l} \epsilon_U\text{-LIP} \\ \epsilon_L\text{-LIP} \end{array} \right\} \Rightarrow$$

$$\boxed{(\epsilon_U + \epsilon_L)\text{-LDP}}$$

$$\epsilon\text{-LDP} \Rightarrow \underline{\epsilon\text{-DP}}$$

$$\epsilon\text{-LDP} \Rightarrow \epsilon\text{-LIP} \quad \textcircled{1}$$

$$e^{-\epsilon} \leq \frac{P(y|s)}{P(y|s')} \leq e^{\epsilon} \Rightarrow e^{-\epsilon} \leq \frac{P(s|y)}{P(s'|y)} \leq e^{\epsilon}$$

$P(S)$

$P(S)$

ϵ -LIP

\Rightarrow ϵ -MI

(2)

ϵ -LDP

\Rightarrow ϵ -MI

(3)

$\left\{ \begin{array}{l} \epsilon$ -DP

~~\Rightarrow~~

ϵ -MI

Counterexample

neighbouring